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Small-Group Learning

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In any math class I've been in before, I just sat and listened to a teacher talk about what was in the book and what would be assigned for homework. When I got to college the same thing was happening except here I take notes on what the professor says. I have never felt in any of these situations that I should express my opinion on the subject. The most any teacher has done to stimulate a discussion on the topic was to simply say 'Questions?' Whenever the teacher said this, though, it didn't sound like he wanted a reply.

Junior Mathematics Major

In a span of less than two years, three national reports [**6**, **9**, **10**] have recommended fundamental changes in the teaching of college mathematics. The most recent document *Moving Beyond Myths* [**10**] states, for example, that "It is widely recognized that lectures place students in a passive role, failing to engage them in their own learning. Even students who survive such courses often absorb a very misleading impression of mathematics—as a collection of skills with no connection to critical reasoning" (p. 24). The document recommends that faculty, among other things, "explore effective alternatives to 'lecture and listen' ", "involve students actively in the learning process," and "teach future teachers in the ways they will be expected to teach" (p. 34).

If we take seriously the charge to "teach future teachers in the ways they will be expected to teach," a reading of the *Professional Standards for Teaching Mathematics* [**8**] (which is referred to in *Moving Beyond Myths*) will lead to using small group approaches for at least part of the class time. This document states, "Students learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas. It is a key function of the teacher to develop and nurture students abilities to learn with and from others—to clarify definitions and terms to one another, consider one another's ideas and solutions, and argue together about the validity of alternative approaches and answers . . . " (p. 58).

Reports and recommendations, of course, do not make changes in the classroom. Only teachers doing things differently achieve that. Changing teaching, however, is not easy. There is both *individual and institutional resistance* to change. My own experience with resistance occurred during my first attempt to use a small group approach in a linear algebra class I taught in 1970, my third year as a faculty member. Although the students liked the class I was so afraid that my colleagues would find out what I was doing that I closed the door of the classroom in case any of them walked by. My anxiety caused me to abandon the approach for three years.

At the institutional level, it was not until 1991 that the MAA annual meeting provided a special session devoted to alternatives to the lecture method, although articles [**3**, **11**] appeared in the 70's describing this approach in mathematics courses and an increasing number of studies (see [**4**] for references) showed the effectiveness of small group cooperative learning approaches.

Having overcome, to some degree, my own resistance to pedagogical change, I thought it would be helpful to offer some suggestions to faculty considering

After the activity, I ask them to reflect on the process with the following instructions: Each person tells how they felt when doing the problem alone and as a group. Discuss the differences between individual learning and small group learning.

Another activity that shows students how small groups can enhance learning is Missing Corners. In this activity the students are asked to (individually) write a description of the pattern in FIGURE 3, construct with cubes (or tiles) the next two figures in the pattern, predict the number of cubes in the nth figure and write a justification for their prediction based on the figures. The students are amazed when they arrive at different ways of describing the pattern and justifying the predictions. (This can be followed by examining more complex, even 3-dimensional, patterns.)

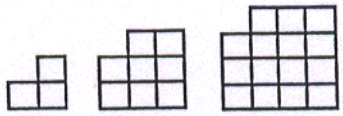

Figure 3

One obstacle when college students begin to work in groups is their lack of experience communicating about mathematics. It is important therefore that early group activities develop communication skills rather than stress solutions or proof. A colleague of mine, Bill Jacob, addresses this issue in a geometry class by having one student draw a geometric figure, a second write instructions on how to draw the figure for a third student, who then draws the figure without having seen the original figure. The figures are compared and the results discussed. Then the roles are rotated. He also has the students write reports on experiments (for example, projective geometry experiments with mirrors).

There are not many examples of college level curriculum written specifically for small group instruction. Two older texts [**5**, **12**] attempt to present traditional course content for a small group approach. There is more available for the pre-college level and these sources may provide ideas for what can be done at the college level. The Interactive Mathematics Project[1] and the California Math A materials[2] are good examples of non-traditional approaches at the secondary level. Bishop [**1**] is a good resource for thinking about how to restructure curriculum for group projects and discussion. I used *Thinking Mathematically* [**7**] successfully in a problem solving course for potential secondary teachers. A good source for reading about what other people have done is [**4**]. Be aware, however, that some of the authors in this book have a very traditional view of mathematics and there is considerable disagreement about classroom practices as well.

It may be necessary to change your ideas of "covering" curriculum. It will help to reflect on the questions: what does it mean to teach? what does it mean to learn? College faculty need to think about and discuss the relative value of exposing students to mathematical knowledge or having them actually do mathe-

[1]This project is developing a three year problem-based mathematics high school mathematics course. Contact Interactive Mathematics Project-EQUALS, University of California, Berkeley, CA 94720.

[2]This material was developed by California secondary teachers to meet the guidelines of the 1985 California Framework. It is being rewritten by Larry Hatfield for publication by Glencoe Publishing Company in June, 1993.

matics. For example, a class for potential secondary teachers explored symmetry by examining some strip patterns from Native American (San Ildefonso Pueblo) pottery. I then asked them to create their own strip patterns with pattern blocks (colored squares, triangles, trapezoids, parallelograms, and hexagons). I then asked the students to classify the strip patterns. With very little help from me most groups (in three to four hours of class time) discovered the seven different classes and were able to justify (although not rigorously) that these were all of them. I could have lectured about it in an hour or two, but I think that the level of understanding would have been shallower. There are no easy answers to questions about breadth versus depth—perhaps no answers at all—but it is beneficial to reflect on and discuss the questions.

STUDENTS. Students will probably be skeptical about participating in a small group at first. You need to explain to them why you are deviating from the traditional lecture method—and remind them periodically of your reasons and "philosophy of education". Some students may continue to struggle with the different approach:

> *Once again as in Math 101A I have mixed feelings arising out of working in groups. One thing, working in small groups tends to make* me *more visible to others. That means my strong points, in between points, and weak points are right there for everyone to see. It is very hard for me to expose my weaknesses to others, i.e., my mistakes. Working in small groups tends to make me confront a feeling of stupidity. It is hard to overcome the urge to compare myself to others and to try to come up with the correct answer. I tend to underplay 'correct' contributions (i.e., a good idea) and overplay any errors I make, so it tends to be a struggle for me.*

Others will make the transition more readily:

> *To be honest, when this class first began I did not enjoy it very much. It is hard for me to pin down why. In part it had a bit to do with the groups. It was not the fact that I did not know my group yet. I knew that we all would get to know one another. It was more because the people in my group seemed to be so much brighter than me. It did not seem as if I would ever have anything worth while to contribute . . . . After a few weeks of classes we all felt comfortable. We not only discussed math topics but also what we did over the weekend. How things were going etc. It was no longer a state of unfamiliarity or any anxiety over making a mistake or saying something foolish . . . [If we had not been in a group] I do not think we would have become friends.*

Many come to understand and value the benefits of small group instruction:

> *Working in small groups is very different than lecturing only. There is no strict relationship where one person knows all the answers (teacher) and the other asks the questions (student). Working in a group is a more equal relationship where hopefully everyone is answering and asking questions. I like working in a small group because it forces you to think rather than just copy whatever the teacher writes.*
>
> *To be honest, in the past the only method that I knew to learn mathematics was to memorize so, therefore, if I memorized the material well I felt pretty good as a learner, but now, however, I realize that I have been somewhat cheated on what and how I learned mathematics. It just seems like I should have a better understanding of what I have learned in the past.*

It is important to pay attention to the quality of the group process. Every three to four weeks I have students assess their group's functioning. I ask them to answer two questions in their weekly journal: What are you doing to contribute to the group's functioning well? What can you do to improve? Then I visit each group and sit down with them and ask each person to talk about their answers. This method provides them time to think about the questions free from pressure, but

ensures that the group is communicating about group processes. It also indicates clearly that I value group process, since I devote my time to assessing it.

Take time to interact with students. They will be uneasy about working in groups and will need time to talk about it. The relationship with the instructor is crucial in making the small group approach work. One student addressed the issue in his journal:

> *I believe that a very useful addition to this course would be to require, perhaps during the second or third weeks, each student to make an appointment during office hours. I believe a one on one discussion on 'what do you want to get out of this course?' to 'what do you want to get out of teaching?' would enhance the entire course. I believe that it would make the students even more aware of what they can get out of the course, as well as, being aware of the usefulness of being available to students for one-on-one talks. Offering offices hours does a lot. However, requiring us to take advantage of office hours would be excellent.*

Be aware of the effect of grading on small groups. The first day (of a course in problem solving for prospective secondary teachers) I told the students how I would grade (see FIGURE 4) and in particular that I did not value memorization but would assess progress in their mathematical reasoning and their ability to communicate (verbally and in writing) about mathematics. I told them that I wanted to try (for the first time) using student portfolios to assess their work.

Attendance	10%;
Contribution to group and class (including an assessment of a portfolio of their work)	20%
Five problem sets	30%
Journal	20%
Final exam (oral)	20%

Figure 4

They were a little uneasy about this, so in the third week of the semester I spoke more about my philosophy, grades, and what portfolios were. I asked them to suggest what kind of evidence would show growth in mathematical thinking. We made a list and I indicated that they should include this type of evidence as part of their portfolio. Because I had devised what I thought would be a very acceptable grading method, and spent some time in class discussing it, I was surprised to read what one student wrote in her journal:

> *The assessment lecture bothered me. Up to that day working in the groups and learning was fun. I had been thoroughly enjoying the class but when the portfolio came up and I realized that something was going to be 'graded' my perspective of the class began to change. All of a sudden I had to pay attention to what I was writing down. 'Is it neat enough?' Am I writing enough?' 'Have I misplaced something that I should have kept?' These questions and slight panic began to be aroused in me. That day our group discussion was much more jumpy and less relaxed. For the first time our ideas came across in a competitive way. I cannot really explain why we became more interested in getting our ideas on paper than playing with the problem. With the knowledge that our progress was going to be measured, our performance became more forced and less enjoyable.*

I have not solved the problem raised by this student. Certainly anxiety about grades is not unique to the small group approach. I have long believed that any "outside" (by someone other than the learner) evaluation of learning interferes with the learning process—with the possible exception of assessment conducted as an integral part of the learning process with the goal of assisting the learner. Furthermore I consistently find that my dual responsibilities of facilitating learning

and evaluating it, are inconsistent. Ideally a learner would be willing to reveal his/her ignorance to a teacher. In reality, he/she may be reluctant to do so to an evaluator. Although the small group approach reduces the interference of grades with learning (for example, anxiety is reduced by talking about grades with friends, students are graded on more than just test results) it does not eliminate it. I am not comfortable with grading and I admit my dilemma to my students. After reading the above student's journal, I read the passage (anonymously) to the class and we discussed grades, competition and learning. It seemed to help.

While on the subject of assessment, it is worth pointing out the obvious. Often students memorize to get by on tests. Success in this system does not necessarily mean that students have learned (understood and are able to use) the mathematics. We often do not notice this when using the lecture method because we only see test results. When you observe students working in groups, however, you will see more clearly what students do and do not understand. It can be disconcerting. In a class on classical number systems, for example, I asked students to use a concrete model to justify the familiar algorithm for adding fractions. They had tremendous difficulty coming up with an explanation.

A final point in regard to your students: Do not be too hard on them. There is an old saying "Don't blame the messenger who brings bad news." In a sense, undergraduates who cannot think or communicate well about mathematics are the message that something is drastically wrong with our education system. Small group instruction is not a panacea. It will not immediately remedy the deficiencies of previous miseducation. But it is a start.

INSTITUTIONAL AND PERSONAL SUPPORT. It will not be easy to give up the lecture method. Both institutional and personal support will be helpful in making the change. The Action Plan of *Moving Beyond Myths* makes many institutional recommendations. Draw these to the attention of relevant officials and organize on your campus for implementing the suggested reforms.

At present there is little opportunity for college faculty to participate in professional development focused on teaching. The educational community regards professional development in both content and methodology as a necessary part of *pre-college* teachers' professional growth. For college instructors, however, professional development focuses on learning more mathematics or on suggested revisions in content, not learning about new pedagogical approaches or research in mathematics education.

Until there is adequate opportunity to participate in professional development activities focused on teaching, individuals will have to strike out on their own. It may be possible to arrange your own professional development by watching someone who is using small groups or participating in a small group experience taught by someone else. In the long run, however, the attitudes and practices within the profession concerning professional development will need to change if large numbers of college faculty are to obtain the support necessary to implement the goals of the reform movement.

Even with institutional support for change you will need to get personal support if you intend to change your teaching. Find people with whom you can discuss mathematics teaching—your ideas, your successes and failures. (Accept that there will be failures.) In addition, find someone who is able to listen to you non-critically. It will be helpful to reflect on what you are doing and deal with your feelings about your efforts without fear of criticism. I did not have that 20 years ago and that is one reason why I abandoned my experimentation for three years. When you

are feeling tense or worried about whether you are doing the right thing you will tend to revert to the 'tried and true.' If you have someone to talk to about your feelings it is more likely that you will be able to think through the issues, and pursue your goals. See [13, 14] for further information about the relationship between feelings, listening and educational change.

CONCLUSION. Teaching using small groups is very different from lecturing. You will need time to develop your abilities. Be prepared for ambivalence and doubts. I encourage you to persist. Virtually every teacher (elementary or secondary) takes mathematics courses in a college or university. How you teach mathematics to undergraduates affects mathematics education throughout the entire system. You can play a crucial role in modeling for future teachers how to teach so that students are actively engaged in doing mathematics. If you are satisfied with large numbers of students not understanding or liking mathematics, with an attrition rate for mathematics students of approximately 50% each year after 9th grade [2], then continue with the lecture method. But if you want to provide opportunities for larger numbers of students to gain deeper understandings and to improve their ability to communicate about mathematics, then explore small group approaches and other alternatives to the lecture method. Perhaps you will be rewarded by having a future secondary teacher write: *I want to implement in my classroom what we did in this class. The most important thing that I have learned is that math can be fun.*

REFERENCES

1. Bishop, A. J., "Mathematical Enculturation: A Cultural Perspective on Mathematics Education." 1988 Kluwer. Dordrecht.
2. Committee on the Mathematical Sciences in the Year 2000. "A Challenge of Numbers." 1990 National Academy Press. Washington, D.C.
3. Davidson, N., The Small Group Discovery Method as Applied in Calculus Instruction. *American Mathematical Monthly*. August–September: 789–91, 1971.
4. Davidson, N., *Cooperative Learning in Mathematics, A Handbook for Teachers*. 1989, Addison Wesley, Menlo Park.
5. Davidson, N. and F. Gulick, "Abstract Algebra: An Abstract Learning Approach." 1976 Houghton Mifflin. Boston.
6. MAA Committee on the Mathematics Education of Teachers. "*A Call for Change: Recommendations For The Mathematical Preparation Of Teachers Of Mathematics*." Leitzel ed. 1991 Mathematical Association of America. Washington, D.C.
7. Mason, J., L. Burton and K. Stacey, "Thinking Mathematically." 1985 Addison Wesley. Reading.
8. National Council of Teachers of Mathematics. "Professional Standards for Teaching Mathematics." 1991 NCTM. Reston.
9. National Research Council. "Everybody Counts: A Report to the Nation About the Future of Mathematics Education." 1989 National Academy Press. Washington, D.C.
10. National Research Council. "Moving Beyond Myths: Revitalizing Undergraduate Mathematics." 1991 National Academy Press. Washington, D.C.
11. Weissglass, J., Small Groups: An Alternative to the Lecture Method. *Two Year College Mathematics Journal*. (Now the *College Mathematics Journal*) 7:15–20, 1976.
12. _____, "Exploring Elementary Mathematics: A Small Group Approach for Teaching," 1979 Kendall-Hunt (originally published by W. H. Freeman.)
13. _____, Constructivist Listening for Empowerment and Change. *The Educational Forum*. 54(4):351–370, 1990.
14. _____, Teachers Have Feelings: What Can We do About It? *Journal of Staff Development*. (12(1)):28–33, 1991.

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implementing small group approaches. In a sense, this is the article I wish I had been able to read twenty years ago.

BEGINNING. Do not be afraid to start slowly. It is not necessary to abandon lecturing completely. For some purposes it is a good method. You can combine lecturing, students working in groups, and whole-class discussion in any proportion you desire. One way to start is to have students form a group or pair and discuss how they solved a homework problem. Alternatively, you can pose an open-ended question for them to think about. Have them write about it (with a 'quick write') and then report their initial thinking to their group. Providing students time to think and write individually before sharing in the group is often helpful. Not all students want to start talking right away.

Another workable method is to set aside a portion of class time for students to discuss a concept or work on a problem, an investigation, or a group project. Some, or all, groups can report on their work to the class (either as a progress report or a final report). You can add perspective and background information as needed. In a large class, where it is cumbersome to use groups, you might have the students spend some time working in pairs—discussing a definition, sharing thoughts about a problem, comparing solutions or exploring a concept. Your Teaching Assistant can, with some encouragement from you, use small groups in discussion sections.

In order to ease the transition from lectures, provide an experience early on in the course demonstrating that a small group approach enhances learning in ways that lectures do not. For example, I often begin my class on problem solving with *Counting Squares*. Students are given a problem (FIGURE 1) and asked to work individually.

Counting Squares
(individual)

How many squares are there in the figure below? Be able to defend your answer. Work by yourself.

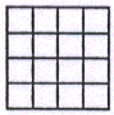

Figure 1

After about 10–15 minutes they are arranged in groups and given the problem in FIGURE 2.

Counting Squares
(small groups)

How many squares are there in the figure below? Work in your groups. Make sure that everyone is able to defend the answer.

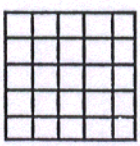

Figure 2